Logarithms Practice - 9/26/16

1. What is $\log_4(16)$?

   Solution: $4^2 = 16$, so $\log_4(16) = 2$.

2. Solve for x: $4\log_3(9x) = 16$.

   Solution: We first divide both sides by 4 to get $\log_3(9x) = 4$. Then to get rid of the $\log_3$, we raise 3 to both sides to get $9x = 3^4$. We then divide by $9 = 3^2$ to get $x = 3^2 = 9$.

3. Solve for x: $\log_2(x) + \log_4(x) = 0$.

   Solution: To get rid of the log, we raise 4 to both sides to get $4^{\log_2(x)+\log_4(x)} = 1$. We can use the exponent rule $a^{xy} = a^x \cdot a^y$ to separate this out into $(2^2)^{\log_2(x)} \cdot 4^{\log_4(x)} = 1$. But $(2^2)^{\log_2(x)} = 2^{2\log_2(x)} = 2^{\log_2(x^2)}$, so we can rewrite this as $2^{\log_2(x^2)} \cdot 4^{\log_4(x)} = 1$. The logs and the exponents cancel, giving us $x^2 \cdot x = 1$, so $x^3 = 1$, so $x = 1$.

4. Solve for x: $3^{2^x} = 9^{4^x}$.

   Solution: Since $9 = 3^2$, we can rewrite as $3^{(2^x)} = 3^{(2\cdot 4^x)}$. Take $\log_3$ of both sides to get $2^x = 2 \cdot 4^x$. Take $\log_2$ of both sides to get $\log_2(2^x) = \log_2(2 \cdot 4^x)$. Using our log rules, we can separate this out as $x\log_2(2) = \log_2(2) + \log_2(4^x)$. We can bring down the x in the last expression to rewrite this as $x\log_2(2) = \log_2(2) + x\log_2(4)$. Since $\log_2(2) = 1$ and $\log_2(4) = 2$, this simplifies to $x = 1 + 2x$. Thus $x = -1$.

5. Solve for x: $\log_4(\log_2(x) + \log_2(8)) = 1$.

   Solution: We start by raising 4 to both sides to get $\log_2(x) + \log_2(8) = 4^1$. Since $\log_2(8) = 3$, we subtract that from both sides to get $\log_2(x) = 4 - 3 = 1$. To get rid of the log, we raise 2 to both sides to get $x = 2$.

6. Solve for x: $\log_{\sqrt{12}}(\log_2(64)\log_3(x)) = 2$.

   Solution: To get rid of the $\log_{\sqrt{12}}$, we raise $\sqrt{12}$ to both sides to get $\log_2(64)\log_3(x) = (\sqrt{12})^2 = 12$. Since $\log_2(64) = 6$, we can divide both sides by 6 to get $\log_3(x) = 2$. To get rid of the $\log_3$, we raise 3 to both sides, so $x = 3^2 = 9$.